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Finite-amplitude strain waves in laser-excited plates

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Abstract
The governing equations for two-dimensional finite-amplitude longitudinal strain waves in isotropic laser-excited solid plates are derived. Geometric and weak material nonlinearities are included, and the interaction of longitudinal displacements with the field of concentration of non-equilibrium laser-generated atomic defects is taken into account. An asymptotic approach is used to show that the equations are reducible to the Kadomtsev–Petviashvili–Burgers nonlinear evolution equation for a longitudinal self-consistent strain field. It is shown that two-dimensional shock waves can propagate in plates.

1. Introduction

The investigation of solitary strain wave propagation in solids under intense external exposures (in particular, under laser or electron-beam radiation or under high-speed loading) represents one of the rapidly progressing lines of research in nonlinear wave dynamics [1–9]. The presence of such waves is usually attributed to the balance between dispersion (or dissipation) and nonlinearity. Dispersion in the elastic medium may be caused by its microstructure [1], as well as by the finiteness of the crystal lattice period [10] or the thickness of the sample [3]. Nonlinearity is provided by both the nonlinear dependence of strain on the displacement gradient [8] (geometrical nonlinearity) and by the elastic features of a material (physical nonlinearity).

The study of the behavior of nonlinear strain waves is of importance for the development of both the general theory of nonlinear wave processes and the modern methods of non-destructive testing [3] and determination of the physical properties of materials, including the detection of regions of defect concentration and quality testing of coatings.

The formation of non-equilibrium atomic point defects of the crystal structure (vacancies, interstitials) may occur as a result of the action of intense external energy fluxes (laser and corpuscular radiations) on solids or as a result of mechanical, thermal, and electric treatment of materials. A high concentration of localized defects is a source of internal mechanical stresses [11]. These stresses are caused by the distortion of the crystal lattice near the defects arising as a result of the breaking of atomic bonds. The defects generated in a plate may diffuse and recombine either at various internal inhomogeneities in the bulk of the plate (or emerge at the surface) or with each other (a mutual recombination).

The presence of a high concentration of non-equilibrium atomic lattice defects in the medium and its relation to the elastic strain may affect the propagation of nonlinear elastic disturbances in a condensed medium and produce qualitatively new physical effects. For example, physical nonlinearities caused by atomic defects may lead to the appearance of relaxation components in the elastic parameters of the medium (in both linear and nonlinear elastic moduli). The presence of lattice defects with a finite relaxation rate may give rise to dissipative terms, which are absent in the conventional equations of elasticity theory.

The nonlinear dynamics of longitudinal strain waves in solids without taking into account the interaction with lattice defects was theoretically investigated in [7–9]. In our previous works [12–16], the evolution of solitary strain waves in a condensed medium was considered with allowance for the interaction with laser-induced non-equilibrium atomic lattice defects. In these studies, attention was mostly focused on studying the influence of the strain-induced diffusion, generation, and recombination of defects on the propagation of one-dimensional (1D) elastic strain disturbances and their dispersion and dissipation properties.

Our aim in this paper is to study the two-dimensional (2D) finite-amplitude longitudinal vibrations in an isotropic elastic plate making allowance for the interaction between the strain and non-equilibrium atomic defect concentration fields. A nonlinear evolution equation describing the 2D longitudinal strain waves is derived using an asymptotic approach. It has the form of a Kadomtsev–Petviashvili–Burgers (KPB) equation.

2. Statement of the problem

Let us consider an isotropic nonlinear elastic plate with free lateral surfaces that occupies the region $-\infty < x, y < \infty$, $-h < z < h$, in Cartesian coordinates (x, y, z). Once laser light is absorbed in the plate, local heating will result in the generation of atomic lattice defects. Let $n^{(j)}(x, y, z, t)$ be the concentration of non-equilibrium atomic point defects (PDs; vacancies and interstitials) of the jth-type ($j = V$ for vacancies (V-defects) and $j = I$ for interstitials (I-defects)). Let q and r be the source function and recombination rate of atomic defects, respectively. Let us enter the Cartesian coordinates $\vec{x}(x, y, z)$, so the centroidal plane of a plate is described by the equation $z = 0$ and the lateral surfaces by the equations $z = \pm h$. Assume the displacement vector in the plate is $\vec{u} = (u_1, u_2, u_3)$. The strain field in the nonlinearity plate is defined by Green finite deformation tensor u_{ik} ($i, k = 1, 2, 3$):

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_{ik}} + \frac{\partial u_i}{\partial x_i}\frac{\partial u_i}{\partial x_k}\right).$$

This describes the geometrical nonlinearity, as discussed by Engelbrecht [8]. The interaction of strain and concentration fields occurs through a direct mechanism due to the modulation of the rate of generation (recombination) of defects owing to the deformation potential.

The governing equations of motion are obtained using Hamilton's principle. Indeed, for an adiabatic deformation, the Lagrangian density per unit volume, L, is defined as the difference between the kinetic energy density, W, and U is the density of the potential energy of the elastic continuum with atomic defects [5, 6]. We have

$$L = W - U = \frac{\rho}{2}\left[\left(\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial u_2}{\partial t}\right)^2 + \left(\frac{\partial u_3}{\partial t}\right)^2\right] - U(u_{ik}),$$

where ρ is the density of the plate material at time $t = t_0$. Let us represent U in the form

$$\begin{aligned} &U = U_{\text{elas}} + U_{\text{d}},\\ &U_{\text{elas}} = \tfrac{1}{2}\lambda u_{ll}^2 + \mu u_{ik}u_{ki} + \tfrac{1}{6}\nu_1 u_{ll}^3 + \nu_2 u_{nn}u_{ik}u_{ki} + \tfrac{4}{3}\nu_3 u_{ik}u_{kn}u_{ni},\\ &U_{\text{d}} = -\vartheta^{(mj)} n u_{ll}, \end{aligned} \tag{1}$$

where U_{elas} is the energy density of the elastic continuum with allowance for anharmonicity (λ and μ are the second-order elastic moduli, or the Lamé coefficients, ν_1, ν_2, and ν_3 are the third-order elastic moduli); U_{d} is the energy density corresponding to the interaction of atomic defects with the elastic continuum; $\vartheta^{(mj)} = K\Omega^{(mj)}$ is the defect deformation potential. The dilatation parameter $\Omega^{(mj)}$ characterizes the lattice deformation due to the appearance of a single j-type point defect in the lattice. For v-defects, $\Omega^{(mV)} = -\delta^{(V)} d_0^3 < 0$ (here, the coefficient is $\delta^{(V)} = 0.2$–0.4 and d_0 is the lattice period), whereas, for i-defects, $\Omega^{(mI)} = \delta^{(I)} d_0^3 > 0$ (the coefficient is $\delta^{(I)} = 1.7$–2.2). In the above formula, $K = \lambda + 2/3\mu$ is the bulk modulus. Defects V and I are represented as a substitutional atom whose volume is smaller or greater than the volume of the matrix atoms, respectively. In what follows, we restrict ourselves to a system with a single type of defect and set $n^{(j)}(x, y, z, t) \equiv n(x, y, z, t)$, $\tau_{\text{d}}^{(j)} \equiv \tau_{\text{d}}$, $\vartheta^{(mj)} = \vartheta^{(m)}$, etc in equation (1).

We set to zero the variation of the action functional Φ:

$$\delta\Phi = \delta \int_{t_1}^{t_2} \mathrm{d}t \left[\int_{-h}^{h} \mathrm{d}z \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} L \,\mathrm{d}x\,\mathrm{d}y\right] = 0. \tag{2}$$

The integration in brackets in equation (2) is carried out at the initial time $t = t_0$.

The displacement vector components (u_1, u_2, u_3) in thickness-symmetrical vibrations of the plate and at low frequencies ($\omega h c_t^{-1} < 2$, ω is the angular frequency, and $c_t = \sqrt{\mu\rho^{-1}}$ is the shear linear wave velocity) may be approximated as follows [17]:

$$\begin{gathered} u_1(x, y, z, t) = u(x, y, t), \qquad u_2(x, y, z, t) = v(x, y, t),\\ u_3(x, y, z, t) = z w(x, y, t), \end{gathered} \tag{3}$$

where the displacements $u(x, y, t)$ and $v(x, y, t)$ are in the centroidal plane along the x and y axes, respectively, and $w(x, y, t)$ is the function describing the displacements along the z axis.

In this case the system of three-dimensional (3D) equations of elasticity theory is replaced by a simpler hyperbolic–parabolic system depending on two variables.

For the stress tensor (σ_{ik}) we have

$$\sigma_{ik} = \lambda u_{ll}\delta_{ik} + 2\mu u_{ik} + O(u_{ik}^2) - \vartheta^{(m)} n\delta_{ik}, \tag{4}$$

where σ_{ik} and u_{ik} are the components of stress and strain tensors, respectively; the function $O(u_{ik}^2)$ includes the nonlinear terms describing anharmonicity of elastic displacements. The last term in (4) takes into account the elastic stresses due to the concentration expansion of the medium. The tensor σ_{ik} takes into account both the geometrical and physical (material) nonlinearities.

In the context of the thermal-fluctuation model of PD production, the rate of defect generation from the lattice sites due to laser irradiation is governed by temperature T (or the power density of laser radiation I_{L}) and stresses. Therefore, this rate may vary under the effect of propagating elastic wave, i.e. thermofluctuation-related defects may be generated and annihilate. Strain wave propagation affects the characteristics of the defects. Thus, when the longitudinal strain wave propagates, the formation energy (w_{f}) of PDs changes in the compression and dilatation zones. The renormalized formation energy of PDs can be represented as $\tilde{w}_{\text{f}} = w_{\text{f0}} - \vartheta^{(\text{d})} u_{ll}$ (w_{f0} is the formation energy for the defect in an unstrained crystal, $\vartheta^{(\text{d})}$ is the deformation potential characterizing the variation of the activation energy of formation of defects under the lattice deformation). If there is a deformation-related perturbation of the lattice, not only does the formation energy of defects decrease, but so does the activation energy for the defect migration (w_m): $\tilde{w}_m = w_{m0} - \vartheta^{(m)} u_{ll}$ (w_{m0} is the migration energy of defects in the absence of strain field); this results in an increase in the diffusion coefficient.

The concentration of PDs is dependent on temperature. One thus needs to know how the laser irradiation affects the local temperature of the plate. We will consider here situations where the laser only heats the material (the light energy absorbed by the medium is transformed into heat), and that an equilibrium between laser radiation and the temperature field (T) is reached on timescales much shorter than the characteristic timescale of evolution of the defect density. Typically, the timescale for equilibration between photon absorption and defect generation is of the order of picoseconds, while that for PD diffusion is of the order of microseconds. We also assume that the contribution of thermal strain to deformation fields is negligible compared to lattice dilatation due to PDs, and that phase changes and chemical reactions in the medium are absent.

In this paper we will analyze the problem of strain wave propagation in thin plates irradiated over a large area by CW or pulsed lasers. Then let us consider the situation of uniform irradiation of the plate surface. We will, furthermore, assume that the temperature profile has reached its equilibrium value. Its evolution is sufficiently slow compared to PD generation, and it can be considered as quasistationary. The solution of the heat conduction equation for this case is given by Duley [18].

Modulation of the formation and migration energies brings about the corresponding modulations of the source function (q) and defect recombination rate (r)

$$q = q_0 + q_1 u_{ll}, \qquad r = r_0 + r_1 u_{ll},$$

where q_0 and r_0 are the values of the source function and recombination rate in an unstrained lattice ($u_{ll} = 0$), and $q_1 = \partial q/\partial u_{ll}$, $r_1 = \partial r/\partial u_{ll}$ are the values of their derivatives at $u_{ll} = 0$.

Using the above-mentioned assumptions we can write the following kinetic equation if the inhomogeneous perturbations in the defect density $n_1 = n - n_0$ are slight ($n_1 \ll n_0$, where $n_0 = q_0\tau_d$ is the steady-state uniform distribution of defects):

$$\frac{\partial n_1}{\partial t} = (q_1 - r_1 n_0) u_{ll} - r_0 n_1. \tag{5}$$

The first term in the round brackets on the right-hand side of equation (5) accounts for the contributions to the generation of PDs that refer to the deformation potential. The second term accounts for the stress-induced recombination of defects ($r_1 = r_0\vartheta^{(m)}/k_B T$), and the third term accounts for the loss of atomic defects due to recombination in a stress-free plate ($r_0 = 1/\tau_{d0} = D_{d0}k_d^2$ is the recombination rate at the sinks and the surface, $k_d^2 = \sum_s Z_s\rho_s + 4/h^2$, where ρ_s is the density of sinks, $D_{d0} = D_0 \exp(-w_{m0}/k_B T)$ is the diffusion coefficient of defects, D_0 the constant, k_B is the Boltzmann constant, and τ_{d0} the relaxation time).

The mutual recombination of different types of PDs in the bulk is neglected. For the generation rate of PDs due to laser irradiation, we have the Arrhenius-type relation [11]:

$$q_1 = q_0 \frac{\vartheta^{(d)}}{k_B T}, \qquad q_0 = d_0^3 \omega_0 N_0^2 \exp\left(-\frac{w_{f0}}{k_B T}\right).$$

Here, ω_0 is the atomic vibrational frequency ($\omega_0 \sim 10^{14}$ s^{-1}) and N_0 is the density of lattice sites.

Solving equation (5) with allowance for boundary conditions $n_1(\pm\infty, t) = 0$, we get

$$n_1 = (q_1 - r_1 n_0) \int_{-\infty}^{t} d\zeta\, u_{ll}(\zeta) \exp[(\zeta - t)/\tau_d]. \tag{6}$$

Substitution of (6) into (4) results in the following equation for the condition of the elastic medium:

$$\sigma_{ik} = \lambda u_{ll}\delta_{ik} + 2\mu u_{ik} + O(u_{ik}^2) - \delta_{ik} g_0 \int_{-\infty}^{t} d\zeta\, u_{ll}(\zeta) \exp[(\zeta - \tau)/\tau_d], \tag{7}$$

where $g_0 = q_0\vartheta^{(m)}(\vartheta^{(d)} - \vartheta^{(m)})/k_B T$.

3. Governing nonlinear equations for longitudinal strain waves

Now we can derive the governing equations of motion for an elastic plate. Substituting (1), (3), and (7) into (2) and using Hamilton's variational principle, after a simple calculation we obtain equations for the displacements u and w:

$$\begin{aligned}
\rho\frac{\partial^2 u}{\partial t^2} &= (\lambda + 2\mu)u_{xx} + (\lambda + \mu)v_{xy} + (3\lambda + 6\mu + \beta)u_x u_{xx} \\
&+ \mu u_{yy} + \lambda k_0 w_x + (\lambda + \mu)[v_x v_{xx} + \tfrac{2}{3}h^2 w_x w_{xx} + (u_y^2)_x] \\
&+ \lambda[2v_y v_{xy} + \tfrac{2}{3}h^2 w_y w_{xy} + (v_y u_x)_x + \tfrac{2}{3}h^2 w_x w_{xx} \\
&+ w w_x + k_0(u_x w)_x + k_0 u_{yy} w] + \mu\left[\frac{\partial}{\partial y}(v_x v_y\right. \\
&\left. + \tfrac{1}{3}h^2 w_x w_y + u_x v_x) + (u_y v_x)_x + 2u_x u_{yy}\right] \\
&+ (\nu_1 + 2\nu_2)\frac{\partial}{\partial x}\left[u_x(v_y + k_0 w) + \frac{1}{2}(v_y^2 + k_0 w^2)\right] \\
&+ \left(\frac{1}{2}\nu_2 + \nu_3\right)\left\{\frac{\partial}{\partial x}\left(u_y^2 + v_x^2 + \frac{1}{2}h^2(w_x^2 + w_y^2)\right)\right. \\
&\left. + 2\frac{\partial}{\partial y}\left[u_y(u_x + v_y + k_0 w)\right]\right\} + 2(\nu_2 + \nu_3)\left\{(u_y v_x)_x\right. \\
&\left. + \frac{\partial}{\partial y}\left[v_x(u_x + v_y + k_0 w)\right]\right\} + \nu_1 k_0 \frac{\partial}{\partial x}(v_y w) - 2\nu_3 k_0 \\
&\times \frac{\partial}{\partial y}(v_x w) + (u_{xx} + u_{yy})I_1 + (1 + u_x)\frac{\partial}{\partial x}(I_1) \\
&+ u_y \frac{\partial}{\partial y}(I_1);
\end{aligned} \tag{8}$$

$$\begin{aligned}
\frac{\rho h^2}{3}\frac{\partial^2 w}{\partial t^2} &= -\lambda k_0(u_x + v_y) - (\lambda + 2\mu)k_0 w \\
&\times (k_0 + \tfrac{3}{2}w) + \tfrac{1}{3}\mu h^2(w_{xx} + w_{yy}) + \tfrac{1}{3}h^2(\lambda + 2\mu) \\
&\times \left[\frac{\partial}{\partial x}(w_x u_x) + \frac{\partial}{\partial y}(w_x u_y)\right] + \frac{1}{3}\lambda h^2\left[\frac{\partial}{\partial x}(w_x v_y)\right. \\
&\left. + \frac{\partial}{\partial y}(w_y u_x)\right] + \frac{1}{3}\mu h^2\left[\frac{\partial}{\partial x}(w_y u_y + w_y v_x)\right.
\end{aligned}$$

$$
\begin{aligned}
&+\frac{\partial}{\partial y}(w_x u_y + w_x v_x) + 2\frac{\partial}{\partial x}(w w_x) - 2\frac{\partial}{\partial y}(w w_y) \\
&- w_x^2 - w_y^2\Big] - \tfrac{1}{2}\lambda[k_0(u_x^2 + u_y^2 + v_x^2 + v_y^2 \\
&+ \tfrac{1}{3}h^2(w_x^2 + w_y^2)) + 2w(u_x + v_y)] + \tfrac{1}{2}\beta k_0^2 w^2 \\
&- (\nu_1 + 2\nu_2)\left[k_0^2 w(u_x + v_y) + \tfrac{1}{2}k_0(u_x^2 + v_y^2)\right] \\
&+ \frac{1}{3}h^2\left(\tfrac{1}{2}\nu_2 + \nu_3\right)\Big\{2\frac{\partial}{\partial x}\left[w_x(u_x + v_y + k_0 w)\right] \\
&+ 2\frac{\partial}{\partial y}\left[w_x(u_x + v_y + k_0 w)\right]\Big\} \\
&- 3\frac{k_0}{h^2}(u_y^2 + v_x^2) - k_0(w_x^2 + w_y^2) \\
&- 2\nu_2 k_0 u_y v_x - \nu_1 u_x v_y - (k_0 + w)I_1 \\
&+ \frac{\partial}{\partial x}(w_x I_2) + \frac{\partial}{\partial y}(w_y I_2) \qquad (9)
\end{aligned}
$$

where the following designations are used

$$
\begin{aligned}
I_1(x, y) &= -b_0(2h)^{-1}\int_{-h}^{h} \mathrm{d}z \int_{-\infty}^{t} \mathrm{e}^{-r_0(t-\tau)} u_{ll}(\tau)\,\mathrm{d}\tau, \\
I_2(x, y) &= -b_0(2h)^{-1}\int_{-h}^{h} z^2\,\mathrm{d}z \int_{-\infty}^{t} \mathrm{e}^{-r_0(t-\tau)} u_{ll}(\tau)\,\mathrm{d}\tau
\end{aligned} \qquad (10)
$$

(the subscripts x and y in equations (8) and (9) indicate derivations with respect to the corresponding variables; $\beta = \nu_1 + 6\nu_2 + 8\nu_3$; $k_0^2 = \pi^2/12$ is a correction factor [4]).

The equation for v can be obtained from equation (9) by replacing u with v, v with u, and x with y, y with x. Equations (8) and (9) represent the integro-differential equations in which the occurrence of integrated terms is caused by defect–strain interaction. These equations without the integral terms represent the well-known equations of the theory of longitudinal nonlinear vibration of thin elastic plates, considered in [3].

Equations (8) and (9) form a closed system. The latter completely describes the dynamics of 2D strain-related perturbations in a plate with defect generation; these perturbations are caused by nonstationary and nonuniform distributions of the PD subsystem. The inverse effect, i.e. variation in the concentration field of atomic defects in a plate as a result of perturbations of elastic strains, is also accounted for.

4. Kadomtsev–Petviashvili–Burgers evolutionary equation for longitudinal strain waves

We consider the dynamics of a long longitudinal strain wave of small but finite amplitude (the wavelength of the propagating wave (Λ) considerably exceeds the amplitude of the vibrations $\Lambda \gg a$). Entering the small parameter describing nonlinearity of the wave process:

$$
\varepsilon = \frac{a(\beta + 3\lambda + 6\mu)}{\Lambda(\lambda + 2\mu)} \ll 1,
$$

where a is the amplitude of the vibrations and Λ is the wavelength.

To simplify the study we replace integrated operators in (10) by differential ones. Expanding the function $u_{ll}(\tau)$ into a Taylor series on power $t - \tau$ and retaining the first two terms in this expansion ($r_0 t \gg 1$), we obtain

$$
\begin{aligned}
I_1(x, y) &= \frac{g_0}{2hr_0}\left[-\int_{-h}^{h} u_{ll}\,\mathrm{d}z + \frac{1}{r_0}\frac{\partial}{\partial t}\left(\int_{-h}^{h} u_{ll}\,\mathrm{d}z\right)\right], \\
I_2(x, y) &= \frac{g_0}{2hr_0}\left[-\int_{-h}^{h} u_{ll} z^2\,\mathrm{d}z + \frac{1}{r_0}\frac{\partial}{\partial t}\left(\int_{-h}^{h} u_{ll} z^2\,\mathrm{d}z\right)\right].
\end{aligned}
$$

Let us take into account these approximation expressions in equations (8) and (9) and analyze the received equations by a asymptotic method.

For further analysis we introduce dimensionless variables

$$
\begin{aligned}
&u^* = \frac{u}{a}, \qquad v^* = \frac{v}{a}, \qquad w^* = \frac{w}{h}, \\
&\xi = \frac{x}{\Lambda} - \frac{c}{\Lambda}t, \qquad \eta = \sqrt{\varepsilon}\frac{y}{\Lambda}, \qquad \chi = \varepsilon\frac{x}{\Lambda}.
\end{aligned} \qquad (11)
$$

The choice of the variables in (11) reflects the different scales of variation of the wave parameters along the x and y axes. Physically it means that because of nonlinearity and dispersion, a disturbance propagating with velocity c along the x axis slowly evolves in the longitudinal (x) and transverse (y) directions.

We seek a solution of the problem in the form of asymptotic expansions in the small parameter (ε) (the asterisks on the appropriate dimensionless variables are omitted for simplicity):

$$
\begin{aligned}
u &= u_0 + \varepsilon u_1 + O(\varepsilon^2), \\
v &= \sqrt{\varepsilon}(v_1 + \varepsilon v_2 + O(\varepsilon^2)), \\
w &= w_0 + \varepsilon w_1 + O(\varepsilon^2).
\end{aligned} \qquad (12)
$$

Equations (8) and (9), except for ε, contain also two small parameters

$$
\varepsilon_1 = h^2(\lambda + \mu)/3\Lambda^2(\lambda + 2\mu) = O(\varepsilon),
$$

$$
\varepsilon_2 = g_0 c/r_0^2\Lambda = O(\varepsilon),
$$

where the parameter ε_1 characterizes the dispersion induced by motions normal to the centroidal plane of the plate, and the parameter ε_2 takes into account the interaction of the strain field with the subsystem of defects due to deformational potential.

Substitution of expressions (12) in the set (8) and (9) leads to an infinite system of equations. In the leading order in ε, we have:

$$
\rho c^2 u_{0\xi\xi} = (\tilde{\lambda} + 2\mu)u_{0\xi\xi} + \tilde{\lambda}k_0 w_{0\xi}, \qquad (13)
$$

$$
\tilde{\lambda}u_{0\xi} + (\tilde{\lambda} + 2\mu)k_0 w_0 = 0. \qquad (14)
$$

From equation (14) we find the relationship between the longitudinal and normal components of the strain:

$$
w_0 = -\frac{\tilde{\lambda}}{k_0(\tilde{\lambda} + 2\mu)}u_{0\xi}, \qquad \tilde{\lambda} = \lambda(1 - g_0/\lambda r_0). \qquad (15)
$$

After substitution of equation (15) into equation (13) we obtain the following expression for the longitudinal wave velocity in the plate

$$c = \sqrt{\left(\tilde{\lambda} + 2\mu - \frac{\tilde{\lambda}^2}{\tilde{\lambda} + 2\mu}\right)\rho^{-1}}. \quad (16)$$

If g_0 is equal to zero, which means that defect generation is absent, the formula (16) follows a well-known expression for the velocity of longitudinal waves in a plate: $c = [E/\rho(1-\sigma^2)]^{1/2}$ (E and σ are Young's modulus and Poisson's ratio, respectively).

Setting terms with powers ε and $\varepsilon^{1/2}$ equal to zero, we obtain the following system of equations:

$$-\tilde{\lambda}k_0 w_{1\xi} - \frac{\tilde{\lambda}^2}{\tilde{\lambda}+2\mu}u_{1\xi\xi} = 2(\tilde{\lambda}+2\mu)u_{0\xi\chi} + (\tilde{\lambda}+\mu)v_{1\xi\eta} + \mu u_{0\eta\eta} + \tilde{\lambda}k_0 w_{0\chi} + (3\tilde{\lambda}+6\mu+\beta)\varepsilon^{-1}u_{0\xi}u_{0\xi\xi} + \frac{2\mu g_0 c}{3r_0^2 K\Lambda\varepsilon}(2u_{0\xi\xi\xi} - k_0 w_{0\xi\xi}), \quad (17)$$

$$\rho c^2 v_{1\xi\xi} = (\tilde{\lambda}+\mu)u_{0\xi\eta} + \mu v_{1\xi\xi} + \tilde{\lambda}k_0 w_{0\eta} = 0, \quad (18)$$

$$\rho\frac{c^2h^2}{3\varepsilon\Lambda^2}w_{0\xi\xi} = -\tilde{\lambda}k_0(u_{0\chi} + v_{1\eta}) + \frac{\mu h^2}{3\varepsilon\Lambda^2}w_{0\xi\xi} - \tfrac{1}{2}(\tilde{\lambda}+2\mu+k_0^2\beta)\varepsilon^{-1}k_0 w_0^2 - (\nu_1+2\nu_2)\varepsilon^{-1} \times (k_0^2 u_{0\xi}w_0 + 1/2k_0u_{0\xi}^2) - \tfrac{1}{2}\varepsilon^{-1}\tilde{\lambda}(k_0u_{0\xi}^2 + 2w_0u_{0\xi}) - \tilde{\lambda}k_0(u_{1\xi} + k_0w_1) - 2\mu k_0^2 w_1 + \frac{2\mu g_0 c}{3\varepsilon r_0^2 K\Lambda} \times (k_0 u_{0\xi\xi} - 2k_0^2 w_{0\xi}). \quad (19)$$

From equation (18) after integration with respect to ξ, and using equation (15), we have the relationship between the shear strains in the wave: $v_{1\xi} = u_{0\eta}$. We substitute this expression, and also equation (15), into equation (19), which differentiate with respect to ξ. As a result we have:

$$\lambda k_0 u_{1\xi\xi} + k_0^2(\tilde{\lambda}+2\mu)w_{1\xi} = \frac{\tilde{\lambda}h^2(\rho c^2 - \mu)}{3\varepsilon k_0(\tilde{\lambda}+2\mu)}u_{0\xi\xi\xi\xi} - \tilde{\lambda}k_0 u_{0\xi\chi} - \tilde{\lambda}k_0 u_{0\eta\eta} - [\tilde{\lambda}k_0^2\beta + \tilde{\lambda}(\tilde{\lambda}+2\mu) - (\tilde{\lambda}+k_0\nu_1+2k_0\nu_2)(\tilde{\lambda}-2\mu)](\tilde{\lambda}+2\mu)^{-1}\varepsilon^{-1}u_{0\xi}u_{0\xi\xi} + \frac{2\mu g_0 c k_0}{3r_0^2K\Lambda\varepsilon}\left(1 + \frac{2\tilde{\lambda}}{\tilde{\lambda}+2\mu}\right)u_{0\xi\xi\xi}. \quad (20)$$

Equating the left-hand side of equation (17) to the left-hand side of equation (20), multiplied by $-\tilde{\lambda}/k_0(\tilde{\lambda}+2\mu)$, and introducing the designation $e = u_{0\xi}$, we arrive at the equation (called the KPB equation) for the self-consistent strain deformation

$$\frac{\partial}{\partial\xi}(e_\chi + a_1 e e_\xi + a_2 e_{\xi\xi\xi} + a_3 e_{\xi\xi}) = -a_4 e_{\eta\eta}, \quad (21)$$

with coefficients

$$a_1 = (\varepsilon\rho b)^{-1}\{3(\tilde{\lambda}+2\mu) + \beta - \tilde{\lambda}[\tilde{\lambda}k_0^2\beta + \tilde{\lambda}(\tilde{\lambda}+2\mu) - (\tilde{\lambda}+\nu_1k_0+2k_0\nu_2)(\tilde{\lambda}-2\mu)](\tilde{\lambda}+2\mu)^{-2}k_0^{-1}\},$$

$$a_2 = \frac{h^2\tilde{\lambda}^2(c^2-c_t^2)}{3\Lambda^2k_0^2\varepsilon\, b(\tilde{\lambda}+2\mu)^2},$$

$$a_3 = \frac{4g_0c\tilde{\lambda}\mu(3\tilde{\lambda}^2+6\tilde{\lambda}\mu+4\mu^2)}{3r_0^2\Lambda K\varepsilon\rho\, b(\tilde{\lambda}+2\mu)^2},$$

$$a_4 = c^2/b, \qquad b = c_l^2 + c^2, \qquad c_l^2 = (\tilde{\lambda}+2\mu)\rho^{-1}.$$

As can be seen from equation (21), all coefficients ($a_i, i = 1, 2, 3, 4$) depend on the elastic moduli λ, μ and on the properties of the defect subsystem in the medium. It is clear that a_1 is caused by the geometrical and physical nonlinearities of the elastic medium, a_2 and a_3 characterize the dispersion and dissipation, caused, respectively, by the thickness of the plate and the interaction of the elastic strain field of the plate with the atomic defect subsystem.

Equation (21) differs from those obtained in [3] without use of the strain–defect interaction. The presence of the dissipative term $a_2e_{\xi\xi}$ in (21) may significantly change the strain wave behavior in the plate.

If we neglect thickness vibrations of the plate, from equation (21) we get the equations for the propagation of nonlinear waves in an unbounded nonlinear elastic medium with defect generation

$$e_{\chi\xi} + \left(\frac{3}{4\varepsilon} + \frac{\beta}{4\rho c_l^2\varepsilon}\right)(e^2)_{\xi\xi} + a_3 e_{\xi\xi} = -\frac{1}{2}e_{\eta\eta}.$$

Equation (21) admits [2], in particular, a traveling 2D exact solution in the form of shock-wave strain structures. Such structures are possible if the coefficients a_1, a_2 have the same sign, and the coefficients a_1, a_3 have different signs that is possible by the appropriate choice of physical parameters of the medium and the defect subsystem ($q_0, r_0, \vartheta_{\mathrm{m,d}}$). The same structure will arise if the coefficients a_1, a_2 have different signs, and the coefficients a_2, a_3 have the same sign.

Let $g_0 < 0$. Let us also write down the inequalities $a_1 > 0, a_2 > 0, a_3 < 0$. Then the solution of (21) can be written as

$$e(\theta) = \frac{18a_3^2}{25a_1a_2} - \frac{12a_3^2}{25a_1a_2}\tanh^2(\theta/2) - \frac{6a_3^2}{25a_1a_2}\tanh(\theta/2), \quad (22)$$

where,

$$\theta = |k_1|\xi + |k_2|\eta - |\omega|\chi, \qquad \omega = \frac{6a_3^3}{125a_2^2} + \frac{5a_2a_4}{a_3}k_2^2, \qquad k_1 = \frac{a_3}{5a_2},$$

k_2 is a any parameter.

From equation (22) it follows

$$e(\theta\to-\infty) \to 12a_3^2/25a_1a_2, \qquad e(\theta\to\infty) = 0.$$

Returning to a dimensional variable

$$\theta = k_1\xi + k_2\eta - \omega\chi = x + (k_2/k_1)y\sqrt{\varepsilon} - ct - (\omega/k_1)\varepsilon ct,$$

we define that the defect-related contribution to the wave velocity is $(\omega/k_1)\varepsilon$.

Thus, from our analysis it follows that under certain conditions the solution of (21) will look like a shock wave with a monotonic structure. It is clear that the excited shock wave will be a tensile wave ($e > 0$).

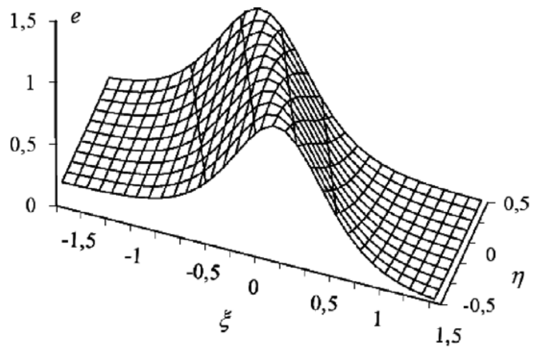

Figure 1. 2D strain waves in laser-irradiated Si plates.

As an example let us consider the formation of shock-wave strain structures in laser-irradiated semiconductor (in particular, Si) plates of thickness $h = 10^{-3}$ cm. To estimate the concentration of generated lattice defects (n_0), we consider here conditions when the duration of a laser pulse (τ_L) exceeds the defect relaxation time (τ_d). In this case the density of defects on the surface of the plate reaches a steady-state value

$$n_0 = q_0\tau_d = q^{(0)}\tau_d^{(0)} \exp(-w_{d0}/k_B T_0),$$

where $w_{d0} = w_{f0} - w_{m0}$, $q_0 = q(T_0)$ is the defect generation rate, T_0 is the steady-state value of the temperature field on the surface, and $q^{(0)}$ and $\tau_d^{(0)}$ are constants. If $I_L = \text{const}$ (uniform irradiation) and in the high surface absorption limit, $\alpha_L h \gg 1$ (α_L is the linear absorption coefficient, $\alpha_L = 10^4\ \text{cm}^{-1}$), for the temperature field, according to [18], we have: $T_0 = I_L(1-R)h/\lambda_T$, where R is the reflectivity coefficient of the plate and λ_T is the thermal conductivity. For $I_L = 6\times10^5\ \text{W cm}^{-2}$, $\lambda_T = 0.25\ \text{W cm}^{-1}\ \text{K}^{-1}$, $R = 0.4$ an estimate

of T_0 yields: $T_0 = 1.5\times10^3$ K. Then, taking $N_0^{-1} = d_0^3 = 2\times10^{-22}\ \text{cm}^3$, $w_{d0} = 1$ eV, a value of $2\times10^{19}\ \text{cm}^{-3}$ may be estimated for the defect concentration (n_0), which is several orders of magnitude less than the concentration of the host atoms. For typical values of the elastic modulus $\lambda = 6.4\times10^{10}$ Pa, $\mu = 7.9\times10^{10}$ Pa, $\beta = 10^{11}$ Pa [22], and the deformation potentials $\vartheta_m = 10$ eV and $|\vartheta_d| = 10^2$ eV [11], estimates of the coefficients (a_1, a_2) and a_3 involved in solution (21) yield: $a_1 = 1.5$, $a_2 = 0.07$ and $|a_3| = 0.2$. We can see from these values that above obtained conditions required for the appearance of a shock-wave strain structure are fulfilled. The shape of the nonlinear wave with the above parameter values is shown in figure 1. Such structures can also be observed in many metal (Al, Fe, Ni, Ti, Mo, etc) plates.

5. Conclusions

A model has been presented for the description of finite-amplitude 2D longitudinal elastic waves in an elastic isotropic plate with non-equilibrium atomic defects. The model is based on the nonlinear equations that uniquely describe the combined dynamics of the fields of longitudinal displacements and concentration of atomic defects. The non-equilibrium concentrations of defects in the plate are caused by the absorption of laser radiation. The interaction of strain and concentration fields occurs through a direct mechanism due to the modulation of activation energies for defect formation and migration owing to the defect deformation potential.

In the long wave limit we derived the nonlinear evolution equation for the 2D strain waves using an asymptotic procedure. The derived nonlinear equation contains a dissipative term caused by the strain–defect interaction. This equation is, in fact, the combinations of the Kadomtsev–Petviashvili equation [19] and the Burgers equation. Here the role of the strain–defect interaction appears similar to the influence of the viscoelasticity on the evolution of strain waves in rods [20].

It is shown that the balance between nonlinearity, dispersion, and defect-related dissipation results in formation in plates of longitudinal solitary waves. The velocity of these waves grows with increase in the wave amplitude, i.e. it depends on a degree of nonlinearity in the wave process. The obtained relationships between the wave characteristics and the geometrical and physical parameters of materials allow us to carry out more correctly experiments on solitary waves in plates with defect generation under laser radiation.

An exact traveling solution describing the 2D solitary strain wave structures (and also the defect concentration wave structures) has been obtained. The analysis of the solution allowed us to conclude that under certain conditions (by the appropriate choice of physical parameters of the elastic medium and the defect subsystem) the longitudinal strain wave can have a shock-wave monotonic structure. The excited shock wave is a tensile wave.

Note that the amplitude and velocity of the solitary strain waves under consideration depend on the elastic moduli and on the properties of the defect subsystem in the plate. Consequently the theory developed here can be used for the determination of the elastic moduli and the parameters of a subsystem of atomic defects (for example, the recombination rate, the migration and formation energies of PDs, and so on) in solids on the basis of nonlinear distortions of the solitary strain wave structures.

To conclude, the model of the evolution of strain waves in solid plates irradiated by laser radiation is of general character and can be applied to other forms of irradiation, such as that with a flux of high-energy particles (electrons, neutrons, etc). It can also be generalized to the problem of propagation of nonlinear longitudinal waves in the plate, initially containing small aggregates (complexes or nanoclusters) of atomic defects. In this case strain-induced generation of atomic defects will occur on a surface of clusters, and their generation rate will depend on the concentration (or radius) of clusters; hence, in order to adequately describe the kinetics of atomic defects we have to supplement equation (5) with the relaxation-type equation for a subsystem of clusters in the form

$$\frac{\partial p}{\partial t} = -\frac{p}{\tau_p^0}\exp(\vartheta_p e/k_B T),$$

where p is the volume fraction of clusters ($0 \leqslant p \leqslant 1$) or the concentration of the complexes; $\tau_p^0 = \tau_p \exp(Q_0/k_B T)$ is the cluster lifetime without taking into account the strain influence, Q_0 is the self-diffusion activation energy for cluster relaxation (bond energy in the case of complexes), τ_p is the decay rate constant, $\vartheta_p = K\Omega_{\rm cl}$ is the deformational potential ($\Omega_{\rm cl}$ is an activation volume of a cluster). For a cluster consisting of N atomic defects, the activation volume $\Omega_{\rm cl}$ is approximately equal to the sum of the activation volume of single atomic defects, and hence, for deformation potential (ϑ_p) we have: $\vartheta_p \approx KN\Omega_{\rm d}$. For example, for a cluster consisting of $N = 10^2$ centers, at typical values of the parameters ($e_0 \approx 10^{-3}$, $\Omega_{\rm d}^{(V)} \approx 10^{-23}$ cm^3, $K = 5 \times 10^{10}$ Pa) we have for the deformation-related contribution to the activation energy the estimation $E_{\rm def} = KN\Omega_{\rm d}e_0 \approx 0.3$ eV. Hence, the energy of elastic concentration stresses is quite sufficient for the detachment of atomic defects from clusters.

An appropriate source term in the kinetic equation for the atomic defects can be represented as:

$$q(e) = -(V_0/\Omega)(\partial p/\partial t)$$

($\Omega \approx d_0^3$ is the atomic volume, V_0 is the initial value of the cluster volume fraction). Such problem was considered in a 1D approximation in [21].

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